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EVALUATION OF KINETIC ENERGY LOSSES IN FLUID FLOW IN TUBES

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It is shown that use of the methods of the thermodynamics of irreversible processes allows one to evaluate the influence of blowing a homogenous gas on the kinetic energy losses in tube flow. Increase of blowing decreases the kinetic energy losses.

Use of the methods of the thermodynamics of irreversible processes gives effective results in the study of heat and mass transfer in homogeneous liquids, solutions and gas mixtures under various conditions. By these methods the most important characteristics of fluid flow in constant and variable-area channels may be established.

On the basis of an analysis of a system of linear Onsager equations in the analytical description of internal friction as transfer of kinetic energy of translational motion and the use of the Curie principle, the authors of [1] developed a theory for the separation of molecular solutions and binary gas mixtures. Experimental data on separation of a solution of NaCl in water, and of a gaseous mixture of nitrogen and oxygen in a rotary separating column under isothermal conditions confirm the theoretical conclusions reached by the authors.

A generalization of experimental data has enabled the optimum conditions of operation of separating columns to be established.

A method of the thermodynamics of irreversible processes was used in [2] to study the flow of a multi-component liquid through a porous medium. It was shown that the influence of friction forces on the flow of a viscous liquid, and the motion of the centers of gravity of the liquid phase, as well as heat conduction and diffusion, cause an increase in entropy whose value may serve as an effective parameter of a system consisting of a liquid and solid phase.

In the present paper, on the basis of the method of thermodynamics of irreversible processes, an evaluation is made of the loss of kinetic energy of a gas moving in a tube into which a homogeneous gas is blown.

It was shown in [3] that a change of state process in a viscous liquid moving continuously in channels may be considered as being in equilibrium, in spite of a finite and arbitrarily large flow velocity. Because of the presence of viscosity and heat conduction in real liquids, this kind of process is accompanied by dissipation of mechanical energy, i.e., the irreversible conversion of mechanical energy into heat, and a decrease of the work capacity of the gas. The amount of mechanical energy irreversibly converted into internal energy of the gas is only part of the mechanical energy converted into heat energy under the influence of forces of surface and internal friction. The other part is again converted into kinetic energy of the flow. The relation between these amounts depends on the flow conditions and may be determined by a thermodynamic analysis of the equilibrium-irreversible flow processes. The second law of thermodynamics, which determines the conditions of conversion of heat into mechanical energy, and, through entropy production, the numerical expression of the degree of irreversibility of real processes, has a particular significance in such an analysis. Friction heat causes an entropy increase in the gas to the same extent as does the heat obtained by the gas as a result of heat exchange with the surrounding medium. Cooling of the gas, on the other hand, produces a decrease in entropy.

In many cases, particularly large flow velocities and small contact times between gas and surface, the heat transfer is comparatively small; the action of friction forces is the most appreciable.

In thermally insulated gas flow, and in cases where the influence of heat transfer on the properties of the moving gas is small, we may consider the entropy increase to be a measure of the loss of translational kinetic energy of the moving gas, due to the action of friction forces. Having determined the increase of entropy of the gas, we may evaluate the corresponding loss of kinetic energy. Therefore, the entropy increment of the flow of a viscous gas in the conditions mentioned is the main thermodynamic property of the flow processes. It enables us to consider as "loss" energy only that portion of the total energy which goes toward increase of molecular or intramolecular motion, and is not capable of subsequent conversion into mechanical work.

The loss of kinetic energy in friction is steady flow of a gas along a cylindrical tube of constant area at given values of initial pressure and back pressure is determined by the product of the absolute temperature T_2 of the gas at the end of the tube and the entropy increment ΔS in the section of the tube examined:

$$\Delta w = T_2 \Delta s. \tag{1}$$

The entropy increment ΔS may be expressed in terms of the heat liberated by the work of friction forces. From an examination of the change of state process of some element of the moving liquid, in a coordinate system moving with it, we may assume that the friction forces acting on the boundaries of the chosen liquid element are external forces with respect to this element. Therefore, the heat liberated due to dissipation of mechanical energy may be considered to be equivalent to the heat obtained externally by the given liquid element. The friction heat causes an entropy increment in the element, as does heat obtained through heat exchange with the surrounding medium or with neighboring liquid elements.

If the mass of the volume of liquid examined is 1 kg, we may write, on the basis of the second law of thermodynamics,

$$ds = dq/T. \tag{2}$$

The action of friction forces tending to slow the motion down causes a pressure drop, whose value in the section dx of a tube of diameter D is

$$dp = \frac{2\,c_f\,u^2}{D}\,\rho\,dx. \tag{3}$$

A decrease of potential pressure energy of the gas of amount $\Delta p/\rho$ corresponds to a pressure drop of amount dp. This energy is converted into internal energy of the gas (heat of friction). Therefore,

$$dq_f = \frac{2\,c_f}{D}\,u^2\,dx. \tag{4}$$

In (3) and (4), the flow velocity u is assumed to be constant over the tube cross section, which does not occur in the general case. Therefore, in place of velocity u constant over the tube, a mean velocity u_m should be used in these equations

$$ds = \frac{2\,c_f}{D}\,\frac{u_m}{T}\,dx. \tag{5}$$

The entropy increment of 1 kg of gas in a tube of length l is determined from the equation

$$\Delta S = \int_0^l 2\,\frac{c_f}{D}\,\frac{u_m^2}{T}\,dx. \tag{6}$$

In order to integrate (6), we establish the change in mean fluid velocity u_m along the tube, using the momentum equations.

The Reynolds equations for time-averaged velocity components, in cylindrical coordinates with origin at the center of a cross section of the tube, have the form

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial r} = -\frac{1}{\rho}\,\frac{\partial p}{\partial x} - \frac{1}{\rho}\,\frac{1}{r}\,\frac{\partial}{\partial r}(\tau r), \tag{7}$$

$$u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial r} = -\frac{1}{\rho}\,\frac{\partial p}{\partial x} + \frac{1}{\rho}\,\frac{\partial r}{\partial x}, \tag{8}$$

where

$$\tau = -\mu\,\frac{\partial u}{\partial r} - \rho\,\overline{u'\,v'}\,. \tag{9}$$

We assume a linear distribution of transverse velocity component v and shearing stress τ over the section of the tube:

$$v = -v_w\,r/R\,. \tag{10}$$

$$\tau = \tau_w\,r/R\,, \tag{11}$$

Averaging the values in (7) over the tube section, and taking (10) and (11) into account, we obtain

$$v_m = -\frac{1}{2}\,v_w; \quad \left[-\frac{1}{\rho}\,\frac{1}{r}\,\frac{\partial}{\partial r}(\tau r)\right]_m = \frac{c_f}{R}\,u_m^2; \quad \left(\frac{\partial u}{\partial r}\right)_m = \frac{u_m}{R}\,.$$

Equation (7) for the flow averaged over the tube section may be written as follows:

$$u_m \frac{du_m}{dx} + \frac{v_w}{2R} u_m = -\frac{1}{\rho}\frac{dp}{dx} + \frac{c_f}{R} u_m^2 . \tag{12}$$

The mean velocity in the tube section is calculated from

$$u_m = \frac{2}{R^2}\int_0^R u\, r\, dr. \tag{13}$$

In [4] an expression was obtained from equations (7)–(9) for the distribution of longitudinal velocity component over the radius of the tube in the form

$$\frac{u_c - u}{u_p} = \frac{1}{\varkappa}\left(\ln\frac{1+\sqrt{r/R}}{1-\sqrt{r/R}} - 2\sqrt{r/R}\right) - \frac{1}{4\varkappa^2}\frac{v_w}{u_p}\times$$
$$\times\left(\ln\frac{1+\sqrt{r/R}}{1-\sqrt{r/R}} - 2\sqrt{r/R}\right)^2, \tag{14}$$

where

$$u_p^2 = -\left(\frac{R}{2\rho}\right)\left(\frac{\partial p}{\partial x}\right).$$

We now obtain an expression from (13) for the mean velocity over the tube section

$$u_m = u_c - \frac{16}{15}\frac{u_p}{\varkappa} + 0.541\frac{v_w}{\varkappa^2}. \tag{15}$$

Equation (12) is reduced to a linear differential equation of the first order in $y = u_m^2$

$$y' + f(x)\, y + g(x) = 0, \tag{16}$$

where

$$f(x) = \frac{1}{R}(2c_f - j);\quad g(x) = -\frac{2}{\rho}\frac{dp}{dx};\quad j = \frac{v_w}{u_m};\quad y' = \frac{d}{dx}u_m^2 .$$

Solving (16) for the unknown function u_m, and determining the constant of integration from the condition that when $x = 0$, $u_m = u_{mi}$, we obtain

$$u_m = \exp\left[-\frac{1}{2R}\int(2c_f - j)\,dx\right]\left\{u_{mi}^2 - \int\frac{2}{\rho}\frac{dp}{dx}\times\exp\left[\frac{1}{R}\int(2c_f - j)\,dx\right]dx\right\}^{1/2}. \tag{17}$$

The change of relative velocity $\bar{u}_m = u_m/u_{mi}$ in terms of the dimensionless coordinate $\bar{x} = x/D$ is given by the equation

$$\bar{u}_m = \exp\left[-\int(2c_f - j)\,d\bar{x}\right]\left\{1 - \int\frac{2}{\rho u_{mi}^2}\frac{dp}{d\bar{x}}\times\right.$$
$$\left.\times\exp\left[2\int(2c_f - j)\,d\bar{x}\right]d\bar{x}\right\}^{1/2}. \tag{18}$$

The entropy increment is determined from the expression obtained for the distribution of mean velocity along the tube

$$\Delta s = \int_{\bar{x}_i}^{\bar{x}}\frac{2c_f}{T}\exp\left[-2\int(2c_f - j)\,d\bar{x}\right]\left\{u_{mi}^2 - \int\frac{2}{\rho}\frac{dp}{d\bar{x}}\times\right.$$
$$\left.\times\exp\left[2\int(2c_f - j)\,d\bar{x}\right]d\bar{x}\right\}d\bar{x}. \tag{19}$$

In the absence of a cross supply of mass, which corresponds to flow in a tube with impermeable walls, the parameter j, which is a measure of the influence of mass transfer through the tube walls, is zero. The solution of (16) has the form

$$u_{mo} = \exp\left[-\frac{1}{2R}\int 2c_{fo}\,dx\right]\left\{u_{mi}^2 - \int \frac{2}{\rho}\frac{dp}{dx} \times \exp\left[\frac{1}{R}\int 2c_{fo}\,dx\right]dx\right\}^{1/2}. \tag{20}$$

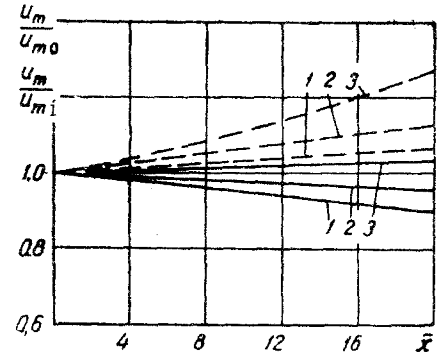

Fig. 1. Chance of velocity along tube: 1) j = 0.001; 2) 0.002; 3) 0.004; solid line—according to (22); broken line—according to (18).

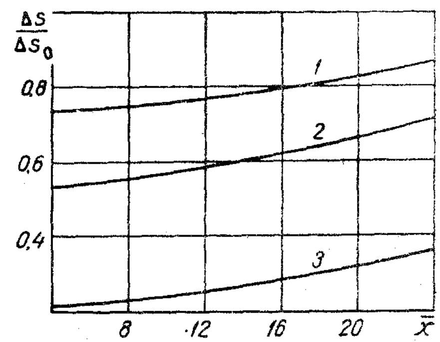

Fig. 2. Entropy increment along tube according to (23): 1, 2, 3) see Fig. 1.

Replacing the dimensional coordinate x in (20) by the dimensionless $\bar{x} = x/D$, and substituting the expression for u_{mo} into (6), we obtain a formula for the entropy increment in isothermal flow in a tube with impermeable walls:

$$\Delta s_0 = \int_0^{\bar{x}} \frac{2c_{fo}}{T} \exp\left[-2\int 2c_{fo}\,d\bar{x}\right]\left\{u_{mi}^2 - \int \frac{2}{\rho}\frac{dp}{dx} \times \exp\left[2\int c_{fo}\,d\bar{x}\right]d\bar{x}\right\}d\bar{x}. \tag{21}$$

Figure 1 shows the change of relative velocity, as determined from (18), at various levels of blowing. It is clear that the velocity drop along the length is the less, the greater the level of blowing, and, at large flow rates of blown gas, the velocity even increases along the length. This follows from the continuity equation.

A comparison is made on the same graph of the velocity along the tube in the presence and absence of blowing, which from (17) and (20) is determined by the relation

$$\frac{u_m}{u_{mo}} = \exp\left[\int 2c_{fo}\,d\bar{x} - \int (2c_f - j)\,d\bar{x}\right] \times \left(\left\{1 - \int \frac{2}{\rho u_{mi}^2}\frac{dp}{d\bar{x}} \exp\left[\int (4c_f - 2j)\,d\bar{x}\right]d\bar{x}\right\} \times \left\{1 - \int \frac{2}{\rho u_{mi}^2}\frac{dp}{d\bar{x}} \exp\left[\int 4c_{fo}\,d\bar{x}\right]d\bar{x}\right\}^{-1}\right)^{1/2}. \tag{22}$$

It is clear from the graph that when gases are blown through the porous wall, the flow in the tube becomes more stable, and the loss of kinetic energy along the tube decreases.

Figure 2 shows a comparison of the entropy increment along the tube in the presence and absence of blowing, which from (19) and (21) is determined by the relation

$$\frac{\Delta s}{\Delta s_0} = \int_0^{\bar{x}} c_f \exp\left[-\int (4c_f - 2j)\, d\bar{x}\right] \times$$

$$\times \left\{1 - \int \frac{2}{\rho u_{mi}^2} \frac{dp}{d\bar{x}} \exp\left[\int (4c_f - 2j)\, d\bar{x}\right] d\bar{x}\right\} d\bar{x} \times$$

$$\times \left\{\int_0^{\bar{x}} c_{fo} \exp\left(-\int 4c_{fo}\, d\bar{x}\right) \times\right.$$

$$\left.\times \left[1 - \int \frac{2}{\rho u_{mi}^2} \frac{dp}{d\bar{x}} \exp\left(\int 4c_{fo}\, d\bar{x}\right) d\bar{x}\right] d\bar{x}\right\}^{-1}. \quad (23)$$

It is clear from the graph that the entropy increment along the tube is the less, the greater the level of blowing. To find the energy balance, it is necessary, of course, to take account of the energy expended in blowing gas through the porous wall.

In drawing the graphs, values of the local friction factor c_f in the conditions examined, at various blowing levels, were calculated according to the method described in [6]; the local friction factor c_{f0} for isothermal flow in an impermeable tube was determined from [7].

The initial air flow velocity was assumed to be 100 m/sec, and the air density corresponded to a temperature of 300° K.

Notation:

dq—amount of heat received by 1 kg of fluid from neighboring elements due to heat conduction dq' and liberated due to dissipation of mechanical energy by friction dq_f; $\varkappa$—constant in the Prandtl mixing length formula; x, r—longitudinal and transverse coordinates; u, v—longitudinal and transverse velocity components; ρ—flow density. Subscripts: w—parameters at wall; o—parameters at an impermeable wall; c—parameters on tube axis.

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